Quantification of Magnetic Moment with MRI

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Introduction

Present in elder populations or patients with dementia, cerebral microbleeds and small calcifications can be identified as small dark round spots in MR images. We can use the complex MR signals around these objects in equations derived from the magnetostatic theory to determine the center of the object and then solve for the magnetic moment. We call this the CISSCO (Complex Image Summation around a Spherical or Cylindrical Object) method. Furthermore, we can quantify the susceptibility difference if the volume of the object can be measured from spin echo images.

By writing a plugin in Java and C++ for the image-processing program ImageJ, we can successfully quantify the magnetic moment of various MR images with these cerebral microbleeds and small calcifications. ImageJ allows processing with the signals in the images to achieve this. We name this plug-in Calculate Magnetic Moment 3D (CMM3D).

Estimating the Spin Density ρ_0 and the Background Phase

In Dr. Cheng's 2015 paper *Magnetic moment quantifications of small spherical objects in MRI*, we have two equations for magnetic moment that can be used to solve for the spin density:

$$Re(f_{ij}) \equiv Re\left(\frac{9\sqrt{3}}{4\pi\rho_0}(S_i - S_j)\right) \tag{9}$$

$$Re(f_{ij}) = \frac{p}{\phi_i}\int_1^{\frac{\phi_j}{\phi_i}}\frac{dx}{x^2}[2\cos(x\phi_i)+\cos(2x\phi_i)] + \int_{-1}^{2}\frac{dx}{x^2}\left[2-(2-x)\sqrt{1+x}\right]\left[\frac{p}{\phi_i}\cos(x\phi_i) - \frac{p}{\phi_j}\cos(x\phi_j)\right] \tag{10}$$

$$where\ \phi_i \equiv \frac{p}{R_i^3}\ with\ i = 1,2,3$$

$$and\ Re(S_i)\ is\ the\ sum\ of\ the\ real\ part\ of\ the\ signal\ in\ the\ radius\ R_i$$

Once the sums for each spherical shell and the phase values at each radius are obtained, we can solve for ρ_0.

At the same time, CMM3D can estimate the background phase of the images. MR images have a background phase that must be removed for accurate calculations. Although a rough estimation of the background phase is done in earlier steps for the rough estimations, we can get a more accurate result using the fact that:

$$e^{i*background\ phase} \propto f_{23} * S_{23}$$

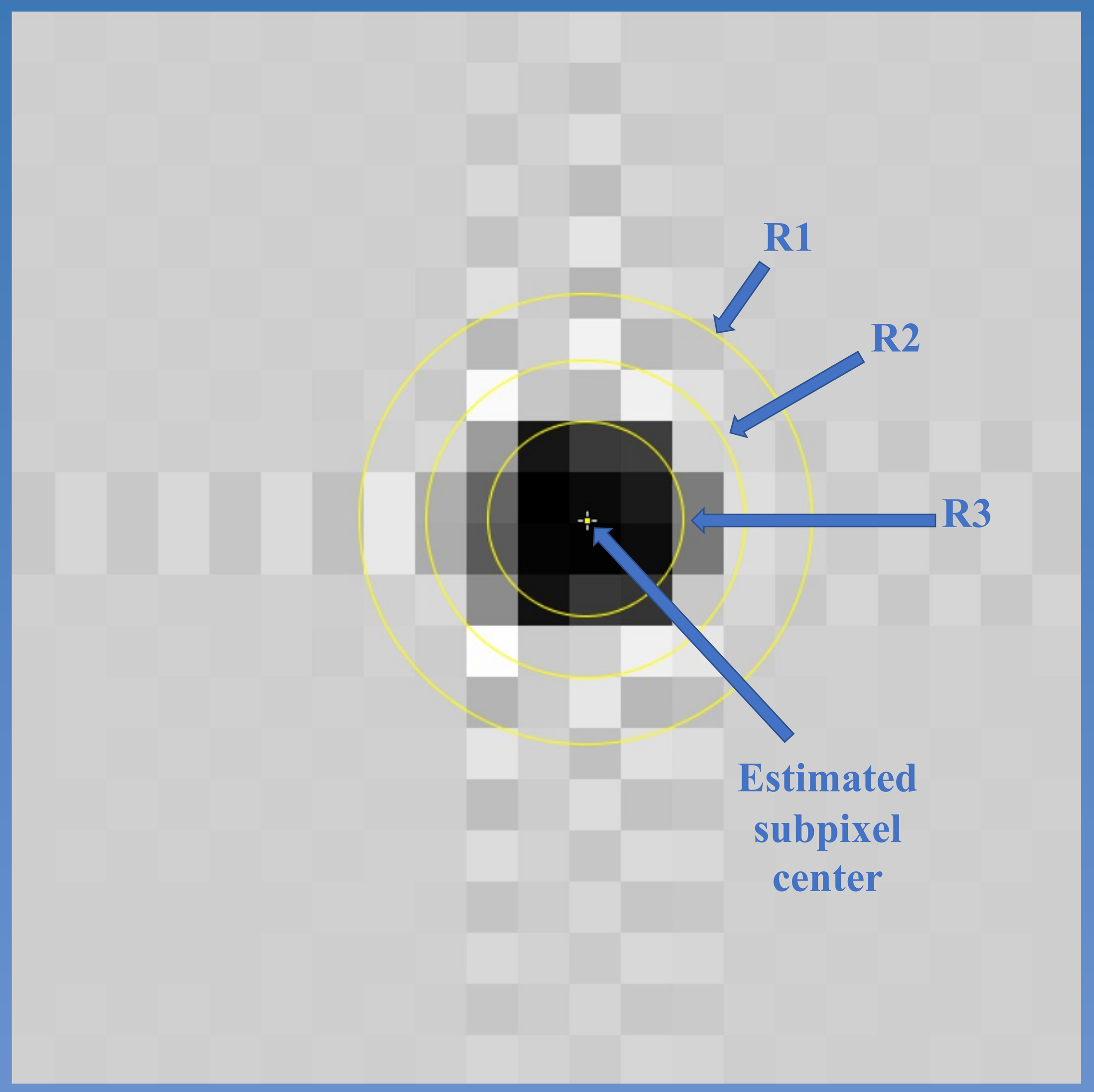

Figure 4: Three selected radii along with the subpixel center

Estimating the Center, Radius, and Magnetic Moment

Looking at Figure 1, one can roughly guess where the center of the object is in the magnitude image. The user will be prompted to draw a rectangular ROI (region of interest) around the object. CMM3D will estimate the center of the object and display it to the GUI using the minimal sum of each plane in the drawn ROI.

Once we have a center, we can estimate the magnetic moment. This is found by using the fact that phase values decrease on the MRI field axis from the center by a factor $\frac{2}{R^3}$ with R being the distance from the center. Looking on the MRI field axis for a phase value of ~ 1, we can use linear interpolation between pixels to get an estimate of the magnetic moment. We can estimate the magnetic moment using:

$$p \equiv \phi R^3$$

Where p is the magnetic moment and ϕ is the phase value at R. We can now use the estimated magnetic moment and phase value and the equation above to find an estimate radius of the object.

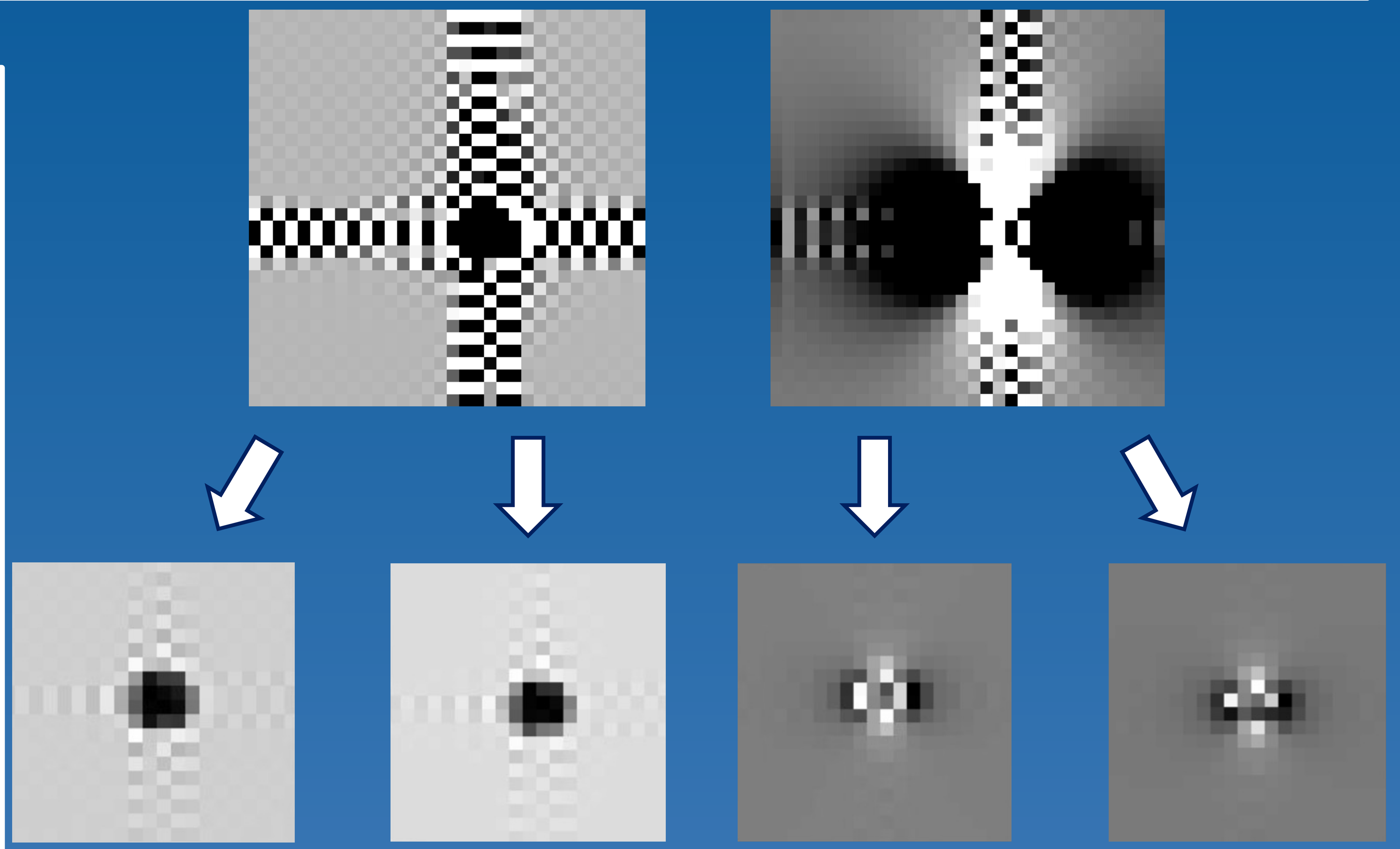

Figure 1: Magnitude (top left) and phase (top right) MR images with their respective subpixel images

Calculating the Magnetic Moment and its Uncertainty

We now have values for a center, spin density, and three radii with respective phase values. We have enough information to quantify our magnetic moment and its uncertainty using:

$$Re(S_1 - S_2)Re(f_{23}) - Re(S_2 - S_3)Re(f_{12}) = 0 \tag{8}$$

We can solve for the magnitude of the magnetic moment p using equation (8) and (9) in CMM3D, since S_i is just the sum of the complex signals within it's respective radius R_i. The program uses Brent's method to find the roots and solve for p.

Equation (9) and (10) are used for our uncertainty calculation. We first obtain a theoretical answer in equation (10). We obtain a real answer from equation (9), except our signal quantities are taken from a set of simulated images generated in Python. These images are generated with the same parameters we have calculated throughout CMM3D. The error between the theoretical and real answer is written as ε_{ij} and is used in the uncertainty calculations:

$$\frac{\delta p}{|p|} = \frac{1}{|D|}\sqrt{(\delta[Re(S_2 - S_3)])^2\frac{\left(Re(f_{12})\right)^2}{p^2} + (\delta[Re(S_1 - S_2)])^2\frac{\left(Re(f_{23})\right)^2}{p^2}} \tag{12}$$

$$where\ D = Re(S_1 - S_2)Re\frac{\partial f_{23}}{\partial p} - Re(S_2 - S_3)\frac{Re\partial f_{12}}{\partial p} \tag{13}$$

$$and\ \left(\partial\left[Re(S_i - S_j)\right]\right)^2 = \left[\varepsilon_{ij}Re(S_i - S_j)\right]^2 + \frac{4\pi}{3}\sigma^2(R_i^3 - R_j^3)\Delta x\Delta y\Delta z \tag{15}$$

Getting a more Precise Center with Subpixel Images

We now have a rough estimation of the center of the object along with its radius. We can then generate our subpixel images around the object center. We do this by taking a square area with twice the estimated radius as the dimensions and divide the voxels in this area into 10^3 sub-voxels.

Using these images, we can estimate the center with more precision using the Amoeba function (or better known as the Nelder-Mead method). This is a direct search method that can find a minimum/maximum in multiple dimensions, which in our case is the center of the object. The division of the voxels to sub-voxels allows the Amoeba search to give more precise results.

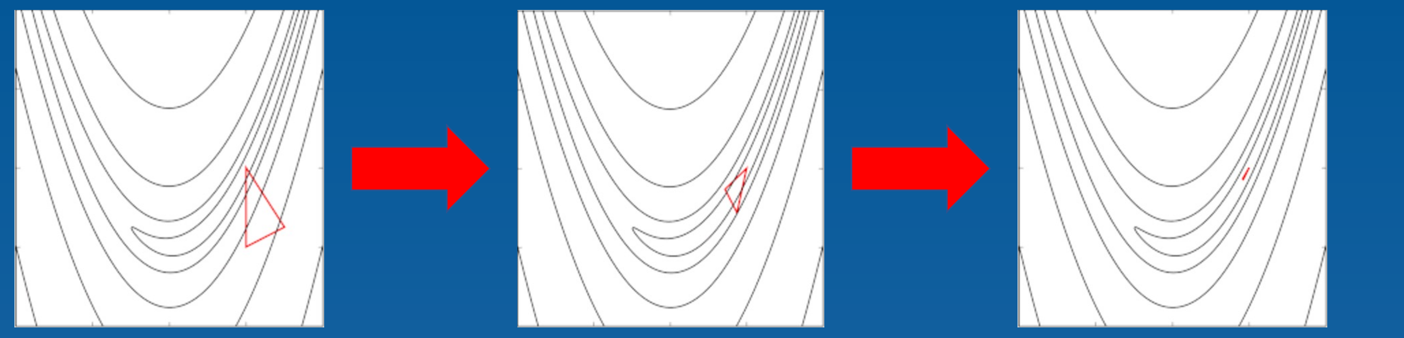

Figure 2: Example of Amoeba search

CMM3D also allows us to study the subpixel phase values in each direction:

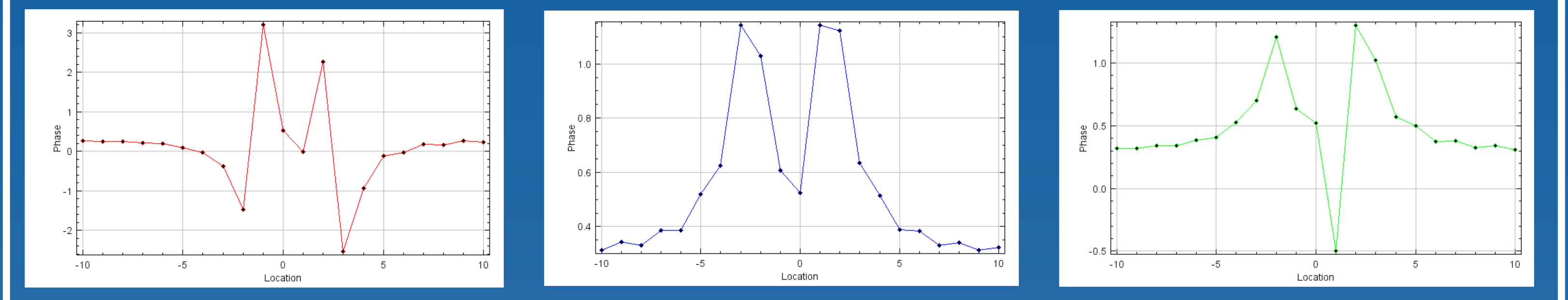

Figure 3: Subpixel phase values in each axis (x, y and z respectively)

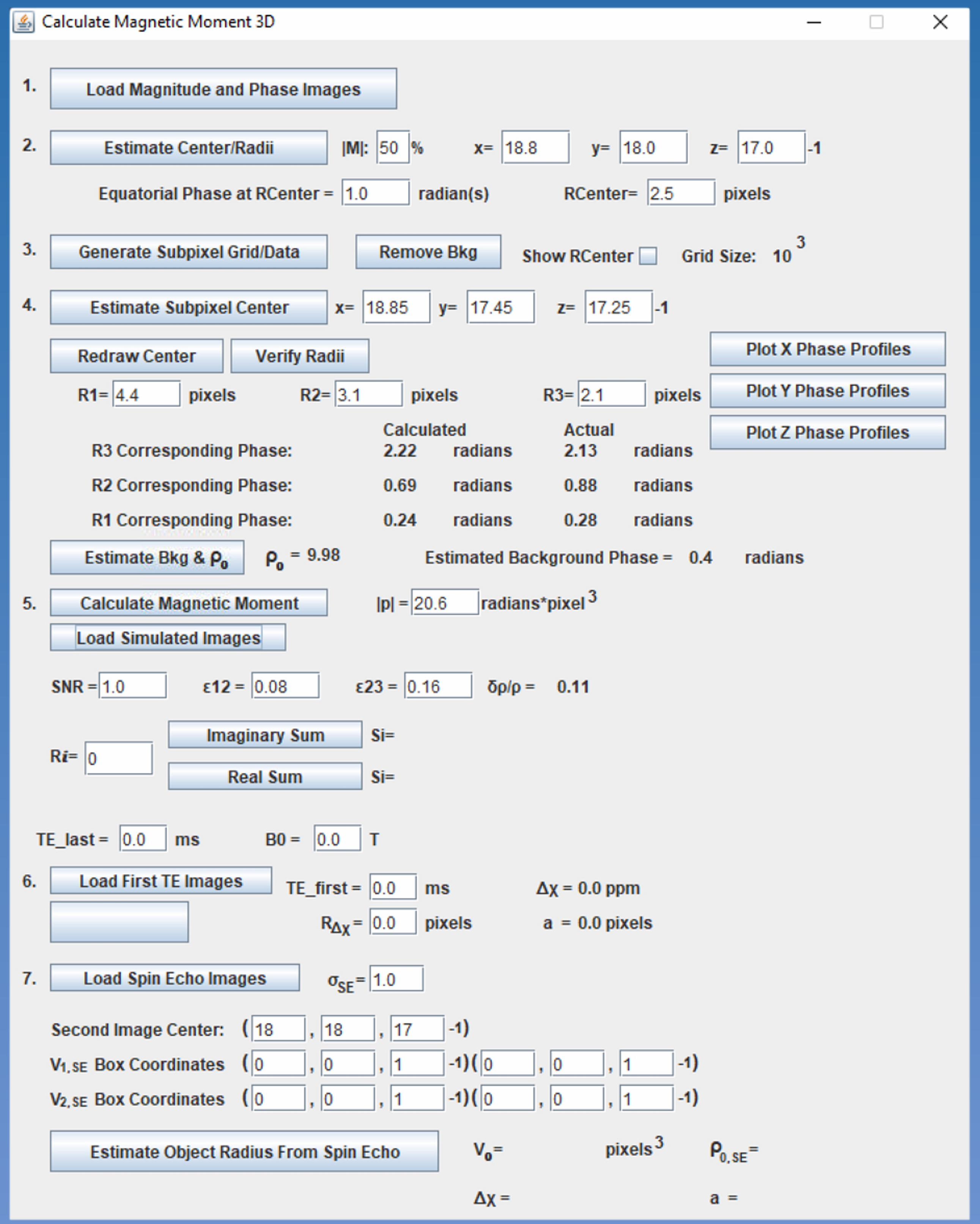

Figure 5: CMM3D GUI

Conclusions and Future Work

Conclusions:

- Magnetic moment was quantified within simulated images
- Magnetic moment uncertainty was quantified

Future work:

- Finish the implementation of step 6, which involves using images at different echo times to estimate the radius more accurately, along with the magnetic susceptibility difference $\Delta\chi$ between the susceptibility of the object and the surrounding material.
- Finish the implementation of step 7, which involves using spin echo images to calculate the volume of the object V_0 and the effective spin density $\rho_{0,SE}$.

References

[1] *Magnetic Moment Quantifications of Small Spherical Objects in MRI*. Cheng, Yu-Chung, et al., Elsevier, July 2015.